Chapter 10 – Counting Methods

Solutions to Exercise 10A

1 **a** $8 + 3 = 11$

b $3 + 2 + 7 = 12$

c $22 + 14 + 1 = 37$

d $10 + 3 + 12 + 4 = 29$

2 **a** $3 \times 4 \times 5 = 60$ meals

b $10 \times 10 \times 5 = 500$ meals

c $5 \times 7 \times 10 = 350$ meals

d $8^3 = 512$ meals

3 Four choices of entrée, eight of main course and four of dessert.

a $4 \times 8 \times 4 = 128$ meals

b $128 +$ no entrée:$128 + 8 \times 4 =$ 160 meals

4 $3 + 7 + 10 = 20$ choices

5 S_1: $2 \times M, 3 \times L, 4 \times S = 9$ choices
S_2: $2 \times H, 3 \times G, 2 \times A = 7$ choices
Total choices $= 9 \times 7 = 63$ choices

6 A to B: 3 airlines or 3 buses
A to S: 4 airlines $\times$ 5 buses.
Total choices $= 3 + 3 + 4 \times 5 = 26$

7 $5(C) \times 3(T) \times 4(I) \times 2(E) \times 2(\text{A}) = 240$ choices

8 Possible codes $= (26)(10^4) = 260000$

9 No. of plates $= (26^3)(10^3) = 17\,576\,000$

10 2 (dot or dash) + 4 (2 digits) + 8 (3 digits) + 16 (4 digits) = 30

Solutions to Exercise 10B

1 **a** $3\,! = 3 \times 2 \times 1 = 6$

b $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

c $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

d $2! = 2 \times 1 = 2$

e $0! = 1$

f $1! = 1$

2 **a** $\frac{5!}{3!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} = (5)(4) = 20$

b $\frac{9!}{7!} = (9)(8) = 72$

c $\frac{3!}{0!}! = \frac{6}{1} = 6$

d $\frac{8!}{6!} = (8)(7) = 56$

e $\frac{5!}{0!}! = \frac{120}{1} = 120$

f $\frac{10!}{7!} = (10)(9)(8) = 720$

3 $5! = 120$ ways

4 $7! = 5040$ ways

5 $4! = 24$ ways

6 $6! = 720$ ways

7 $\frac{10!}{7!} = 720$ ways

8 $\frac{8!}{5!} = 336$ ways

9 TROUBLE:

a All letters used $= 7! = 5040$ ways

b Three letters only $= \frac{7!}{4!} = 210$ ways

10 PANIC:

a All letters used $= 5! = 120$ ways

b Four letters only $= \frac{5!}{1!} = 120$ ways

11 COMPLEX:

a No re-use: $\frac{7!}{3!} = 840$ ways

b Re-use: $7^4 = 2401$ ways

12 NUMBER:

a No re-use: $\frac{6!}{3!}$(3-letter) + $\frac{6!}{2!}$(4-letter)
$= 120 + 360 = 480$ codes

b Re-use: $6^3 + 6^4 = 1512$ codes

13 $\epsilon = \{3, 4, 5, 6, 7\}$, no re-use:

a $\frac{5!}{2!} = 60$ 3-digit numbers

b Even 3-digit numbers: must end in 4 or 6, so 2 possibilities only for last digit. 4 possibilities then for lst digit and 3 for 2nd digit.
$\therefore\ 4 \times 3 \times 2 = 24$ possible even numbers.

c Numbers > 700:
3-digit numbers must begin with 7
$\therefore\ 4 \times 3 = 12$
4-digit numbers: $\dfrac{5!}{1!} = 120$
5-digit numbers: $5! = 120$
Total = 252 numbers

c >7000: 4-digit numbers must begin with 7 or 8: $2 \times \dfrac{5!}{2!}! = 120$
5-digit numbers: $\dfrac{6!}{1!} = 720$
6-digit numbers: $6! = 720$
Total = 1560 numbers

14 $\epsilon = \{3, 4, 5, 6, 7, 8\}$, no re-use:

a 2-digit + 3-digit: $\dfrac{6!}{4!} + \dfrac{6!}{3!} = 150$

b 6-digit even: $3 \times 5! = 360$

15 4 boys, 2 girls:

a No restrictions: $6! = 720$ ways

b 2 ways for girls at end $\times 4!$ for boys $= 48$ ways

Solutions to Exercise 10C

1 a $(V, C), (V, S), (C, S) = 3$

b $(J, G), (J, W), (G, W) = 3$

c $(T, W), (T, J), (T, P), (W, J), (W, P), (J, P) = 6$

d $(B, G, R), (B, G, W), (B, R, W), (G, R, W) = 4$

2 a $^5C_3 = \dfrac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 2 \times 1} = 10$

b $^5C_2 = \dfrac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 3 \times 2 \times 1} = 10$

c $^7C_4 = \dfrac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1 \times 3 \times 2 \times 1} = 35$

d $^7C_3 = \dfrac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 4 \times 3 \times 2 \times 1} = 35$

a = **b**, **c** = **d**

3 a $^7C_3 = \dfrac{20 \times 19}{2} = 190$

b $^{100}C_{99} = 100$

c $^{100}C_2 = \dfrac{100 \times 99}{2} = 4950$

d $^{250}C_{248} = \dfrac{250 \times 249}{2} = 31\,125$

4 a $\binom{6}{3} = \dfrac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 3 \times 2 \times 1} = 20$

b $\binom{7}{1} = \dfrac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{1 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = 7$

c $\binom{8}{2} = \dfrac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$
$= 28$

d $\binom{50}{48} = \dfrac{50 \times 49}{2} = 1225$

5 $\binom{13}{7} = \dfrac{13 \times 12 \times 11 \times 10 \times 9 \times 8}{6 \times 5 \times 4 \times 3 \times 2 \times 1} =$
1716

6 $\binom{25}{3} = \dfrac{25 \times 24 \times 23}{3 \times 2 \times 1} = 2300$

7 $\binom{52}{7} =$
$\dfrac{52 \times 51 \times 50 \times 49 \times 48 \times 47 \times 46}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$
$= 133\,784\,560$

8 $\binom{45}{6} = \dfrac{45 \times 44 \times 43 \times 42 \times 41 \times 40}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$
$= 8\,145\,060$

9 $\binom{3}{4}\binom{4}{2} = \left(\dfrac{3 \times 2 \times 1}{1 \times 2 \times 1}\right)\left(\dfrac{4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1}\right)$
$= 18$

10 a $\binom{30}{8} =$
$\dfrac{30 \times 29 \times 28 \times 27 \times 26 \times 25 \times 24 \times 23}{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$
$= 5\,852\,925$

b Choose 2 men first:
$\binom{10}{2} = \frac{10 \times 9}{2} = 45$

6 women: $\binom{20}{6}$

$= \frac{20 \times 19 \times 18 \times 17 \times 16 \times 15}{720}$

$= 38\,760$

Total $= 45 \times 38\,760 = 1\,744\,200$

11 $2♥: \binom{13}{2} = \frac{13 \times 12}{2} = 78$

$5♠: \binom{13}{5} = \frac{13 \times 12 \times 11 \times 10 \times 9}{120}$

$= 1287$

7-card hands of $5♠, 2♥ = 1287 \times 78$

$= 100\,386$

12 **a** Without restriction:
$\binom{12}{5} = \frac{12 \times 11 \times 10 \times 9 \times 8}{120} = 792$

b $3W + 2M: \binom{8}{3}\binom{4}{2} = (56)(6) = 336$

13 $6F, 5M$, 5 positions:

a $2F + 3M: \binom{6}{2}\binom{5}{3} = (15)(10) = 150$

b $4F + 1M: \binom{6}{4}\binom{5}{1} = (15)(5) = 75$

c $5F: \binom{6}{5} = 6$

d 5 any: $\binom{11}{5} =$
$\frac{11 \times 10 \times 9 \times 8 \times 7 \times 6}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = 462$

e $\geq 4\text{F} = n(4\text{F} + 1\text{M}) + n(5\text{F})$

$= 75 + 6 = 81$

14 $15T, 12F$, 10 selections:

a Unrestricted: $\binom{27}{10} = 8\,436\,285$

b $10T$ only: $\binom{15}{10} = 3003$

c $10F$ only: $\binom{12}{10} = 66$

d $5T{+}5F: \binom{12}{5}\binom{15}{5} = 2\,378\,376$

15 $6F, 4M$, 5 positions:

$3F + 2M \binom{6}{3}\binom{4}{2} = (20)(6) = 120$

$4F + 1M \binom{6}{4}\binom{4}{1} = (15)(4) = 60$

$5F$ only: $\binom{6}{5} = 6$

Total = 186

16 Each of the five times she can choose or refuse
$\therefore$ Total choices $= 2^5 = 32$

17 Total choices

$$= \binom{8}{0} + \binom{8}{1} + \binom{8}{2} + \cdots + \binom{8}{8}$$

$$= 2^8 = 256$$

18 Total colours (cannot choose no colours)

$$= \binom{5}{1} + \binom{5}{2} + \cdots + \binom{5}{5}$$

$$= 2^5 - 1 = 31$$

19 6 fruits, must choose ≥ 2:

$$\text{choices} = \binom{6}{2} + \binom{6}{3} + \cdots + \binom{6}{6}$$

$$= 2^6 - 7 = 57$$

20 6 people, 2 groups:

a n (two equal groups) $= \binom{6}{3} \div 2 = 10$

b n (2 unequal groups) $= \binom{6}{1} + \binom{6}{2} = 21$

Solutions to Exercise 10D

1 $\epsilon = \{1, 2, 3, 4, 5, 6\}$
4 digits, no repetitions; number being even or odd depends only on the last digit.
There are 3 odd and 3 even numbers, so:

a Pr(even) = 0.5

b Pr(odd) = 0.5

2 COMPUTER:
Pr (lst letter vowel) $= \frac{3}{8} = 0.375$

3 HEART; 31etters chosen:

a $\Pr(H\text{1st}) = \frac{1}{5} = 0.2$

b $\Pr(H) = 1 - \Pr(H')$

$$= \left(1 - \left(\frac{4}{5}\right)\left(\frac{3}{4}\right)\left(\frac{2}{3}\right)\right)$$

$$= 1 - \frac{2}{5} = 0.6$$

c Pr (both vowels) $= 3\left(\frac{2}{5}\right)\left(\frac{1}{4}\right) = 0.3$
(Multiply by 3 because the consonant could be in any of the 3 positions.)

4 There are $6! = 720$ ways of filling the 6 seats, but only $2\binom{3}{2}4! = 144$ have end places with women.
$\therefore \Pr = \frac{144}{720} = 0.2$

5 $7W, 6M$, team of 7:
$\binom{13}{7} = 1716$ possible teams

$3W + 4M : \binom{7}{3}\binom{6}{4} = (35)(15) = 525$

$2W + 5M : \binom{7}{2}\binom{6}{5} = (21)(6) = 126$

$1W + 6M : \binom{7}{1}\binom{6}{6} = 7$

658 arrangements with more men than women
$\therefore \Pr = \frac{658}{1716} = \frac{329}{858}$

6 8 possible combinations, so there are a total of $2^8 - 1 = 255$ possible sandwiches.

a $2^7 = 128$ including H
$\therefore \Pr(H) = \frac{128}{255} = 0.502$

b $\binom{8}{3} = 56$ have 3 ingredients
$\therefore \Pr = \frac{56}{255}$

c $\binom{8}{3} + \binom{8}{4} + \binom{8}{5} + \ldots + \binom{8}{8}$
$= 219$ contain ≥ 3 ingredients
$\therefore \Pr = \frac{219}{255} = \frac{73}{85}$

7 $5W, 6R, 7B$, no replacement:

a $\Pr(R, R, R) = \frac{6}{18} \times \frac{5}{17} \times \frac{4}{16} = \frac{5}{204}$

b There are exactly $\binom{18}{15} = 816$ selections.
$\binom{5}{1}\binom{6}{1}\binom{7}{1} = 210$ have all 3 colours.
$\therefore$ Pr (all different colours)
$= \frac{210}{816} = \frac{35}{136}$

8 $5R, 2B, 3G, 4$ picks,
$\binom{10}{4} = 210$ selections:

a $\Pr(G', G', G', G') =$
$\left(\frac{7}{10}\right)\left(\frac{6}{9}\right)\left(\frac{5}{8}\right)\left(\frac{4}{7}\right) = \frac{1}{6}$

b $\Pr(\geq 1G) = 1 - \Pr(\text{No } G) = \frac{5}{6}$

c $\Pr(\geq 1G \cap \geq 1R)$:
$N(G + R + B + B) = \binom{3}{1}\binom{5}{1}\binom{2}{2} = 15$
$N(G + R + R + B) = \binom{3}{1}\binom{5}{2}\binom{2}{1} = 60$
$N(G + G + R + B) = \binom{3}{2}\binom{5}{1}\binom{2}{1} = 30$
$N(G + G + R + R) = \binom{3}{2}\binom{5}{2} = 30$
$N(G + G + G + R) = \binom{3}{3}\binom{5}{1} = 5$
$N(G + R + R + R) = \binom{3}{1}\binom{5}{3} = 30$
Total = 170
$\therefore \Pr(\geq 1G \cap \geq 1R) = \frac{17}{21}$

d $\frac{\Pr(\geq 1R | \geq 1G))}{\Pr(\geq 1G)} = \frac{17}{21} \div \frac{5}{6} = \frac{34}{35}$

9 $\epsilon = \{0, 1, 2, 3, 4, 5, 6, 7\}$
4 four-digit number (with no repetitions) $= \frac{8}{4}! = 1680$ possible numbers, but any beginning with zero must be taken out, and there are $\frac{7}{4}! = 210$ of these.
$\therefore$ 1470 numbers

a,b It is easier to find the probability of an odd number first. Begin with the last digit: 4 odd numbers. Then look at the first digit: cannot have zero, so 6 numbers. Then there are 6 choices for the second digit and 5 choices for the first.
Total choices $= 6 \times 6 \times 5 \times 4 = 720$.
So $\Pr(\text{odd}) = \frac{720}{1170} = \frac{24}{49}$
Then $\Pr(\text{even}) = 1 - \frac{24}{49} = \frac{25}{49}$

c $\Pr(< 4000)$: must begin with 1, 2 or 3.
Since there are no other restrictions,
$\Pr(< 4000) = \frac{3}{7}$

d $\Pr(< 4000 | > 3000) =$
$\frac{\Pr(3000 < N < 4000)}{\Pr(N > 3000)}$
For $N > 3000$ it cannot begin with 1 or 2: $\therefore$ 6 possibilities
3 other numbers are unrestricted
$\therefore$ total $(N > 3000) = 1050$
For $3000 < N < 4000$ it must begin with 3, so $\frac{7!}{4!} = 210$ satisfy this restriction.
$\therefore \frac{\Pr(3000 < N < 4000)}{\Pr(N > 3000)} = \frac{210}{1050}$
$= \frac{1}{5}$

10 52 cards, 5 selections, no replacement:

a $\Pr(A', A', A', A', A') =$
$\left(\frac{48}{52}\right)\left(\frac{47}{51}\right)\left(\frac{46}{50}\right)\left(\frac{45}{49}\right)\left(\frac{44}{48}\right) = 0.659$

b $\Pr(\geq 1\text{A}) = 1 - 0.659 = 0.341$

c $\binom{52}{5}$ hands, $\binom{51}{4}$ contain $A♠$

$\therefore \Pr(A♠)$

$$= \frac{51 \times 50 \times 49 \times 48}{52 \times 51 \times 50 \times 49 \times 48} \div \frac{4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1}$$

$$= \frac{5}{52} \cong 0.096$$

d $\Pr(A♠ | \geq 1A) = \Pr(A♠) \div \Pr(\geq 1A)$

$$= \frac{5}{52} \div 0.341 \cong 0.282$$

(You cannot simply say that the ace is equally likely to be any of the 4 ($\Pr = \frac{1}{4}$) because there could be more than one ace, hence $\Pr > \frac{1}{4}$.)

11 $5W, 4M$, 3 selections, no replacement:

a $\Pr(W, W, W) = \left(\frac{5}{9}\right)\left(\frac{4}{8}\right)\left(\frac{3}{7}\right) = \frac{5}{42}$

b $\Pr(\geq 1W) = 1 - \Pr(M, M, M)$

$$= 1 - \left(\frac{4}{9}\right)\left(\frac{3}{8}\right)\left(\frac{2}{7}\right) = \frac{20}{21}$$

c $\Pr(2M | \geq 1M) = \dfrac{\Pr(2M)}{1 - \Pr(W, W, W)}$

$\Pr(2M)$: $\binom{9}{3} = \frac{9 \times 8 \times 7}{6} = 84$ selections;

$\binom{4}{2}\binom{5}{1} = 30$ have $2M$

$\therefore \Pr(2M) = \frac{30}{84} = \frac{15}{42}$

$\Pr(2M | \geq 1M) = \frac{15}{42} \div \frac{37}{42} = \frac{15}{37}$

Solutions to Exercise 10E

1 **a** $x^4 + 8x^3 + 24x^2 + 32x + 16$

b $16x^4 + 32x^3 + 24x^2 + 8x + 1$

c $16x^4 - 96x^3 + 216x^2 - 216x + 81$

d $27x^3 - 27x^2 + 9x - 1$

e $16x^4 - 32x^3 + 24x^2 - 8x + 1$

f $-32x^5 + 80x^4 - 80x^3 + 40x^2 - 10x + 1$

g $-243x^5 + 405x^4 - 270x^3 + 90x^2 - 15x + 1$

h $16x^4 - 96x^3 + 216x^2 - 216x + 81$

Solutions to Review: Short-answer questions

1 **a** $^{1000}C_{998} = \dfrac{1000 \times 999}{2} = 499\,500$

b $^{1000000}C_{99999} = 1\,000\,000$

c $^{100000}C_1 = 1\,000\,000$

2 Integers 100 to 999 with 3 different digits:
9 ways of choosing the 1st, 9 ways of choosing the 2nd, 8 ways of choosing the 3rd $= 9 \times 9 \times 8 = 648$

3 $1, 2, 3, 4, 5, 6$, 3 digits, no replacement
$= \dfrac{6!}{3!} = 120$

4 n brands, 4 sizes, 2 scents $= 8n$ types

5 9000 integers from 1000 to 9999:
$N(5' + 7') = 7(\text{1st}) \times 8^3(\text{2nd to 4th})$
$= 3584$
$\therefore 9000 - 3584 = 5416$ have at least one 5 or 7

6 $50M, 30W$, choose $2M$, 1 W:
$\dbinom{50}{2}\dbinom{30}{1} = 36750$ committees.

7 Choose 2 V from 5, $2C$ from 21, no replacement:
$\dbinom{5}{2}\dbinom{21}{2} = 2100$ possible choices.
Each can be arranged in 4! ways
$= 2100 \times 24 = 50400$ possible words

8 **a** 3 toppings from 5, no replacement
$= \dbinom{5}{3} = 10$

b 5 toppings can be present or not
$= 2^5 = 32$

9 7 people to be arranged, always with A and B with exactly one of the others between them:
Arrange (A, X, B) in a block of 3. This can be either (A, X, B) or (B, X, A), and X could be any one of 5 other people.
$\therefore$ 10 possibilities for this block.
There are 4 other people, plus this block, who can be arranged in 5! ways.
$\therefore$ Total $N = 10 \times 5!$
$= 1200$ arrangements

10 OLYMPICS: 31etters chosen:

a All letters equally likely
$\therefore \Pr(\text{O, X, X}) = \dfrac{1}{8}$

b $\Pr(\text{Y}') = \dfrac{7}{8}\left(\dfrac{6}{7}\right)\dfrac{5}{6} = \dfrac{5}{8}$
$\therefore \Pr(\text{Y}) = \dfrac{3}{8}$

c $N(\text{O} \cap \text{I})$ has 3! arrangements of O, I, X
$\Pr(\text{O, I, X}) = \dfrac{1}{8}\left(\dfrac{1}{7}\right)\dfrac{6}{6} = \dfrac{1}{56}$
$\therefore$ Pr (both chosen) $= \dfrac{6}{56} = \dfrac{3}{28}$

11 The coefficients in the 7th row of Pascal's triangle are: 1, 6, 15, 20, 15, 6, 1

$$(x-1)^6$$
$$= \sum_{i=0}^{i=6} \binom{6}{i} x^{6-i}(-1)^i$$
$$= \binom{6}{0} x^6(-1)^0 + \binom{6}{1} x^5(-1)^1 + \binom{6}{2} x^4(-1)^2 + \binom{6}{3} x^3(-1)^3$$
$$+ \binom{6}{4} x^2(-1)^4 + \binom{6}{5} x^1(-1)^5 + \binom{6}{6} x^0(-1)^6$$
$$= 1 \times x^6 - 6 \times x^5 + 15 \times x^4 - 20 \times x^3 + 15 \times x^2 - 6 \times x^1 + 1 \times x^0$$
$$= x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1$$

Solutions to Review: Multiple-choice questions

1 E $\binom{8}{1}\binom{3}{1}\binom{4}{1} = 96$

2 D $\binom{3}{1}M \times (\binom{5}{1}L + \binom{3}{1}S) = 24$

3 A 10 people, so possible arrangements $= 10!$

4 D 2 letters, 4 digits, no replacement:
$= (\frac{26!}{24!})(\frac{10!}{6!}) = 3276000$

5 C ${}^{21}c_3 = \frac{21!}{3!18!}$

6 B 52 cards, 6 chosen, no replacement:
${}^{52}c_6$

7 C 12 DVDs, 3 chosen, no replays:
${}^{12}c_3 = 220$

8 A $10G, 14B, 2$ of each:
${}^{10}C_2 \times {}^{14}c_2$

9 E METHODS:
Pr(vowel 1st) $= \frac{2}{7}$

10 E $4M, 4F$, choose 4:
$\binom{8}{4} = 70$ teams

$N(3W, 1M) = \binom{4}{3}\binom{4}{1} = 16$

$\therefore \Pr(3W, 1M) = \frac{16}{70} = \frac{8}{35}$

Solutions to Review: Extended-response questions

1 **a** $1, 2, 3, 4, \ldots, 9$ Even digits: $2, 4, 6, 8$ Odd digits: $1, 3, 5, 7, 9$

		4 ways ↓	3 ways ↓			2 ways ↓	1 way ↓	
O	E	O	E	O	E	O	E	O
↑ 5 ways	↑ 4 ways			↑ 3 ways	↑ 2 ways			↑ 1 way

The multiplication principle gives
$5 \times 4 \times 4 \times 3 \times 3 \times 2 \times 2 \times 1 \times 1 = 2880$ ways.

b The digits 1 and 2 are considered as a block.
The block can be organised in $2! = 2 \times 1$ ways, i.e.   12   21
There are 8 objects to arrange [12] 3 4 5 6 7 8 9
$\therefore$ $8!$ arrangements
Total number of arrangements $= 8! \times 2! = 80640$.

2 **a** Three people can be seated in $10 \times 9 \times 8 = 720$ ways in 10 chairs.

b Two end chairs can be occupied in $3 \times 2 = 6$ ways. This leaves 8 chairs for the remaining person to choose from, i.e. $6 \times 8 = 48$ ways of choosing a seat.

c If two end seats are empty it leaves 8 chairs to occupy: $8 \times 7 \times 6 = 336$ ways

3 **a** The odd digits are $1, 3, 5, 7, 9$.
The number three-digit numbers which can be formed $= 5 \times 4 \times 3 = 60$,
i.e. there are 60 different three-digit numbers formed from the odd digits, using each digit only once.

b The numbers greater than 350.
Case 1: Consider numbers greater than or equal to 500.
The first digit can be chosen in 3 ways (it must be 5, 7 or 9). The second digit can be chosen in 4 ways, and the third digit in 3 ways.
There are $3 \times 4 \times 3 = 36$ such numbers.
Case 2: Consider numbers greater than 350 but less than 500.
The first digit can be chosen in one way. It must be 3. The second digit can be chosen in 3 ways (it must be 5, 7 or 9) and the third digit in 3 ways.
There are $1 \times 3 \times 3 = 9$ numbers.
Therefore a total of 45 numbers are greater than 350.

4 **a** There are ${}^{10}C_4 = 210$ ways of choosing a committee of 4 from 10 people.

b there are two men and two women to be selected, there are ${}^5C_2 \times {}^5C_2$ ways of doing this. That is, there are 100 ways of forming the committee.

c When a man is on a committee his wife can't be on it.

Method 1

- If there are 4 men on the committee, then the condition is satisfied.
  There are 5 ways of choosing such a committee.
- If there are 4 women on the committee, then the condition is satisfied.
  There are 5 ways of choosing such a committee.
- If there are three men on the committee, then the other place can be chosen in 2 ways.
  There are $2 \times {}^5C_3 = 2 \times 10 = 20$ ways having this situation.
- If there are two men on the committee, then the other two places can be chosen in ${}^3C_2$ ways.
  There are ${}^5C_2 \times {}^3C_2 = 10 \times 3 = 30$ ways having this situation.
- If there is one man on the committee, then the other three places can be chosen in ${}^4C_3$ ways.
  There are ${}^5C_1 \times {}^4C_3 = 5 \times 4 = 20$ ways having this situation.

Therefore the total number of ways = 5 + 5 + 20 + 30 + 20 = 80.

Method 2

Another way of considering this is with order.
The first person can be chosen in 10 ways.
The second in 8 (as the partner is also ruled out).
The third in 6 and the fourth in 4 ways.
This gives $10 \times 8 \times 6 \times 4 = 1920$ ways, but here order has been considered, and so divide by $4! = 24$ to give $\frac{1920}{24} = 80$.

5 **a** There are ${}^{15}C_4 = 1365$ ways of selecting the batteries.

b There are ${}^{10}C_4 = 210$ ways of selecting 10 charged batteries.

c
$$\begin{aligned}\text{Having at least one flat battery} &= \text{total number} - \text{none flat}\\ &= 1365 - 210\\ &= 1155\end{aligned}$$

6 **a** There are $^{18}C_4 = 3060$ ways of selecting the lollies.

b There are $^{11}C_4 = 330$ ways of choosing the lollies with no mints.

c There are $^{11}C_2 \times 7C_2 = 1155$ ways having two mints and two jubes.

7 **Division 1**

The number of ways of choosing 6 winning numbers from 45

$$= {}^{45}C_6$$
$$= 8\,145\,060$$

$$\therefore \quad \text{probability of winning Division } 1 = \frac{1}{8\,145\,060}$$
$$= 1.2277\ldots \times 10^{-7}$$
$$\approx 1.228 \times 10^{-7}$$

Division 2

There are 6 winning numbers, 2 supplementary numbers, and 37 other numbers
∴ number of ways of obtaining 5 winning numbers and a supplementary

$$= {}^{6}C_5 \times {}^{2}C_1 \times {}^{37}C_0$$
$$= 6 \times 2$$
$$= 12$$

$$\therefore \quad \text{probability of winning Division } 2 = \frac{12}{8\,145\,060}$$
$$= 1.4732\ldots \times 10^{-6}$$
$$\approx 1.473 \times 10^{-6}$$

Division 3

Number of ways of obtaining 5 winning numbers and no supplementary

$$= {}^{6}C_5 \times {}^{2}C_0 \times {}^{37}C_1$$
$$= 6 \times 37$$
$$= 222$$

$$\therefore \quad \text{probability of winning Division } 3 = \frac{222}{8\,145\,060}$$
$$= 2.7255\ldots \times 10^{-5}$$
$$\approx 2.726 \times 10^{-5}$$

Division 4

Number of ways of obtaining 4 winning numbers

$$= {}^6C_4 \times {}^{39}C_2$$
$$= 15 \times 741$$
$$= 11\,115$$

$$\therefore \quad \text{probability of winning Division } 4 = \frac{11115}{8\,145\,060}$$
$$= 0.001\,364\,6\ldots$$
$$\approx 1.365 \times 10^{-3}$$

Division 5

Number of ways of obtaining 3 winning numbers and at least one supplementary

$$= {}^6C_3 \times 2C_1 \times 37C_2 + {}^6C_3 \times 2C_2 \times 37C_1$$
$$= 20 \times 2 \times 666 + 20 \times 37$$
$$= 27\,380$$

$$\therefore \quad \text{probability of winning Division } 5 = \frac{27\,380}{8\,145\,060}$$
$$= 0.003\,3615\ldots$$
$$\approx 3.362 \times 10^{-3}$$

8 a Spot 6

The number of ways of selecting 6 numbers from 80

$$= {}^{80}C_6$$
$$= 300\,500\,200$$

20 numbers are winning numbers

The number of ways of selecting 6 numbers from 20

$$= {}^{20}C_6$$
$$= 38760$$

$$\therefore \quad \text{probability of winning with Spot } 6 = \frac{38\,760}{300\,500\,200}$$
$$= 1.2898\ldots \times 10^{-4}$$
$$\approx 1.290 \times 10^{-4}$$

b Spot5

The probability of winning with Spot = $\dfrac{{}^{20}5C_5}{{}^{80}C_5}$

$$= \frac{15504}{24\,040\,016}$$

$$= 6.4492\ldots \times 10^{-4}$$

$$\approx 6.449 \times 10^{-4}$$